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On the Initial Morphology of Density Perturbations

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Abstract. The morphological distribution of primordial density peaks is assessed. Previous determinations, those of Peacock & Heavens (1985) and those of Bardeen *et al.* (1986) have contradictory concluded that there exist a tendency towards prolate or towards oblate shapes, respectively. By using two methods, the Hessian and the inertia tensor momenta we have performed numerical determinations of the triaxiality of density perturbations in Gaussian random fields with power law spectra. We show that there is no present any tendency of shapes, and that the triaxiality distribution is independent of the spectral index. Moreover, it is shown that the results of Peacock and Heavens are compatible with our determinations. These results are in complete agreement with current triaxial distributions infered for galaxies and clusters both, from observations and from numerical simulations.

Key words: Galaxies, clusters: Gaussian random fields: primordial shapes

1. Introduction

In the extensive analysis of Random Gaussian fields by Bardeen, Bond, Kaiser & Szalay (1986, BBKS), it was found that the density maxima are intrinsically triaxial with some preference towards oblate shapes. Several authors have attempted to connect such a distribution of triaxiality, or the height of the peaks in the primordial density field to the observed distribution of triaxiality of galaxies and clusters. For instance, Dubinski (1992) based on the Peacock & Heavens (1985, PH) results, compared the triaxiality distribution of density peaks to the triaxiality of dark halos in hierarchical models. He aimed to study whether the initial triaxiality distribution, with a tendency towards prolateness, is preserved during the non-linear evolution of the density field. His hypothesis was, that if the shape of peaks indeed determines the shape of dark haloes around galaxies, an excess of prolate dark halos may be expected because of the natural bias for prolate initial conditions, which he did not find. Furthermore, Evrard (1989) and Evrard, Silk & Szalay (1990) tried to explain the Hubble sequence by identifying elliptical galaxies with the highest peaks in the density field, and spirals with the lowest peaks.

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Some other authors have attempted to infer the 3-dimensional shape of galaxies and clusters from the observed projected axial ratios. Fasano & Vio (1991), for example, analysed 204 elliptical galaxies and showed that the distribution of triaxiality of these galaxies is incompatible with both, a distribution of pure oblate or pure prolate spheroids. A very low number of round elliptical galaxies was found in their sample. Lambas, Maddox & Loveday (1992) studied a larger sample of 20399 galaxies, including spiral and ellipticals galaxies. They confirmed that elliptical galaxies are triaxial with no preference for oblate or prolate shapes.

Clusters of galaxies have also been studied by e.g. Carter & Metcalfe (1980), who found not only that clusters of galaxies are flatter than elliptical galaxies, but also that their distribution of shapes is better fitted with a set of oblate spheroids. On the other hand, Binggeli (1982) analysed the apparent ellipticity of 44 Abell clusters, finding that the shapes distribution is consistent with prolate shapes. Furthermore, Plionis, Barrow & Frenk (1991), with a more complete sample of 6000 clusters determined that the morphological distribution is inconsistent with pure oblate objects, and conclude that a triaxial distribution of shapes would fit better their sample. Such a trend has been confirmed by using numerical simulations in hierarchical models by Efstathiou *et al.* (1985) and Frenk *et al.* (1988). Concerning superclusters, there also exist contradictory claims regarding their shapes. West (1989) generated a catalog of 48 probable superclusters by using a large sample of Abell clusters and found that their shape is consistent with prolate objects. Meanwhile, Plionis *et al.* (1992) by employing several statistical techniques and a sample of Abell and ACO (Abell, Corwin & Olowin, 1989) clusters, obtained that superclusters are triaxial with a strong preference toward oblatness. Because the density field is in the linear regimen on large scales, it is possible that the morphology of large-scale perturbations shall reflect the primordial shapes. In adition, there are theoretical evidences in favor that information of the initial shape at the moment of galaxy formation can be preserved after the relaxation process (Aarseth & Binney, 1978).

Clearly, there is no consensus on the distribution of triaxiality for the different observed population of cosmic objects. Whilst several statistical properties of Gaussian and Non-Gaussian statistics have been addressed by e.g. BBKS, Couchmann (1987), Barrow & Coles (1987, 1990) and Coles (1990), concerning shapes of peaks in Gaussian fields there are only two studies with contradictory conclusions. For PH peaks present a tendency towards prolate shapes, whereas the analytical study

by BBKS gave a trend towards oblate shapes. The real shape and triaxial distribution of density perturbations, are highly desirables if one aims to understand the effect of the non-linear evolution of the density field on the morphology of perturbations, and moreover, the effect of tidal interactions. In this paper, we perform numerical determinations of the triaxiality of density peaks and show that Gaussian random fields present no tendency for oblate or prolate shapes. They are intrinsically triaxials. We also confirm the BBKS statement that triaxiality distribution is not sensitive to the spectral index of power law spectra and therefore is useless to constraint cosmological models. For these goals the rest of the paper is organized as follows. In Section 2, we start by briefly review the theretical procedure to assess shapes. A description of the adopted numerical methods is done in Section 3. Section 4, contains our results and a comparison with prior determinations. A discussion and our conclusions are presented in section 5.

2. Shape of Peaks: theoretical formulation

In the following analysis we shall adopt the procedure and notation of BBKS. To first approximation one can expand the density field around a peak as

$$\delta(\mathbf{r}) = \delta(0) + r_i \frac{\partial \delta(\mathbf{r})}{\partial r_i} \mid_{\mathbf{r}=0} + \frac{1}{2} r_i r_j \frac{\partial^2 \delta(\mathbf{r})}{\partial r_i \partial r_j} \mid_{\mathbf{r}=0} + ..., \quad (1)$$

where the second term on the r.h.s. is zero due to the extremum condition. The term composed by the second derivatives of the field is the Hessian matrix

$$H \equiv -\frac{\partial^2 \delta}{\partial r_i \partial r_j}, \quad (2)$$

whose eigenvalues ordered, without lack of generality, as $\lambda_1 > \lambda_2 > \lambda_3$ represent the quadratic approximation

$$\delta(r) = \delta(0) - \sum \lambda_i r_i^2/2, \quad (3)$$

which defines an ellipsoid with semiaxes

$$a_i = [\frac{2\delta(0) - \delta_t}{\lambda_i}]^{1/2}, \quad (4)$$

where δ_t defines the value of the isodensity countour for which the triaxiality is to be determined. Equation (4) is the definition adopted by PH to analyse the morphology of density peaks in terms of the ratio of their main axes.

Aditionally, the triaxiality parameters are defined by BBKS in terms of the Hessian eigenvalues as

$$\epsilon = \frac{\lambda_1 - \lambda 3}{2\sum \lambda_i}, \qquad p = \frac{\lambda_1 - 2\lambda_2 + \lambda_3}{2\sum \lambda_i}, \quad (5)$$

where ϵ measures the ellipticity in the $\lambda_1 - \lambda_3$ plane and p measures the prolatness or oblatness. If $0 \geq p \geq -\epsilon$ then the ellipsoid is prolate-like, while if $\epsilon \geq p \geq 0$ it is oblate-like. The limiting cases are, $p = -\epsilon$ for prolate spheroids and $p = \epsilon$ for an oblate spheroid.

The conditional distribution, $P(\epsilon, p \mid x)$, which describes the probability of finding a density perturbation with triaxiality parameters ϵ and p in the interval $\epsilon + d\epsilon$ and $p + dp$, given the value of the normalized curvature $x = \nabla^2\delta/\sigma_2$ (see eq. [7.6] in BBKS), is

$$P(\epsilon, p \mid x) = (\frac{2}{2\pi)})^{1/2} \frac{x^8}{f(x)} \exp(-(5/2)x^2(3\epsilon^2 + p^2)))W(\epsilon, p), \quad (6)$$

where we have used $x = x^* = \gamma\nu$, the most probable value of the curvature, and γ is a function of the spectral index $\gamma = [(n+3)/(n+5)]^{1/2}$. Hereafter the height of the peaks will be expressed in units of the root mean square σ of the density field as $\delta = \nu\sigma$. Thus, given a probability P and a threshold heigth ν_{max} of the peaks, equation (6) defines an isoprobability curve which encloses at least $100(1-P)\%$ of all the peaks in the density field with height $\nu < \nu_{max}$. Figure 1 exhibits some of the isoprobability contours as a function of the probability and height of the peaks for the spectral indexes $n = 1, -2$. Only these spectral indexes were included to enhance any possible dependence of the triaxiality distribution on the spectrum index. Nevertheless, no extreme dependence is observed. Our numerical results will be compared with the theoretical contours of Figure 1.

3. Numerical Approximation

We have used the Hessian and the inertia tensor methods to assess the intrinsic shape of peaks. THe first one, was used by PH to carry out numerical determinations of the morphology of peaks, whereas the second method was adopted by BBKS for their analytical treatment. Since we aim to numerically test and collate these results, both methods require of smoothing the discrete density field, randomly generated in a 64^3 box, and an interpolation process. We based this latter on the triangular-shaped cloud method (Hockney & Eastwood, 1988). This enables us the reconstruction of the density field at any point within the cubic grid. Further, in order to apply the interpolation, it is also required to set up the minimum number of contributing points-density to the inertia momenta necessary to get stable results of the morphology. In the calculations, we have associated a unit mass to each point of the peak, though various others weighting schemes can be used, see e.g. West (1989b) and Plionis *et al.* (1992). A fully detailed description of the method used, and the determination of the minimum number of grid points and steps of interpolation can be found in González (1994). N_s denotes the number of sub-divisions of each 1^3 cube that is crossed by the isodensity surface.

3.1. Methodology

We generated random fields with a power law spectrum, $P(k) = Ak^n$, with spectral indexes $n = 1, 0, -1, -2$. The 64^3 box used in the calculation has a physical length of $64h^{-1}$Mpc on a side. We then proceed as follows:

1. The density field is convolved with a Gaussian filter
$$W_G = \exp(-(k^2R^2)/2), \quad (7)$$
where k is the perturbation wavenumber and R is the filtering radius.
2. The positions of grid maxima are found, which are defined as those grid points with overdensity δ_{max} higher than the density of their 26 nearest neighbours. In general, the grid maxima positions do not coincide with position of real peaks.
3. The value of the isodensity surface δ_t, for which the triaxiality parameters will be calculated, is chosen by introducing a factor f
$$\delta_t = f\delta_{max}, \quad (8)$$

Fig. 1. Isoprobability contours of the distribution of triaxiality for peaks as a function of their height ν, for $n = 1, -2$. The theoretical 50% isoprobability contours are included, which shall contain half of the total number of peaks.

with $0 << f \leq 1$. Then, the inertia tensor is calculated

$$I_{kl} = \frac{1}{N} \sum_{i=1}^{m} [(x_k - < x_k >)(x_l - < x_l >)], \tag{9}$$

which involves the use of the grid points and the interpolation code of $N_s = 14$ steps, and where $k, l = 1, 2, 3, x_{1i}, x_{2i}$ and x_{3i} are the Cartesian coordiantes of the ith point. For a given f, the sum runs over all the cubic nodes with $\delta \leq \delta_t$. The inertia tensor eigenvalues $a_3^2 > a_2^2 > a_1^2$, are then related to the eigenvalues of the Hessian matrix

$$\lambda_1 = 1/a_1^2, \qquad \lambda_2 = 1/a_2^2 \qquad \text{and} \qquad \lambda_3 = 1/a_3^2. \tag{10}$$

4. In order to assure the validity of the quadratic approximation, the calculation of the Hessian matrix must be made close to the position of the real peak. Therefore, one of our requeriments of the inertia tensor method, is to perform an interpolation process to localize the real maximum. Afterwards, the Hessian matrix is constructed by interpolating the density at points separated at distances $d = R/3, R/2, R$ from the peak center. These three distances were used to test the numerical stability and constancy of the shape around peaks.

Fig. 2. Morphology of peaks as a function of their heigh, for $n = 1$. The theoretical 50% isoprobability contours is included, which contains approximately half of the total number of peaks. The mean triaxiality parameters are $<\epsilon>= 0.174$ and $<p>= 2.3 \times 10^{-2}$.

4. Results

4.1. Statistical analysis

Figure 2 shows the distribution of the triaxiality parameters as a function of the threshold height. It confirms the trend for the highest peaks, $\nu > 3.5$, to be more spherical than the lower ones as found by BBKS. The large dispersion of peaks shapes for all over the $\epsilon - p$ plane, comes from the peaks lower than $\nu < 1.5\sigma$. Meanwhile, the distribution of shapes for peaks in the interval $1.5 < \nu < 3$, –where the number density of peaks is maximum– , is more concentrated towards sphericity and at least 50% of them lay within the theoretical isoprobability contours. If we look at the form of these isoprobability contours, we will notice that the major part of the area bounded by them, is into the oblate shapes side. In order to check whether this should be, or not, understood as a tendency towards oblateness, we have calculated the ratio Γ of the number of oblate ellipsoids N_o to the number of prolate ones N_p, both of which are numerically obtained. Any significant trend must be reflected in Γ. In the second and third columns of Table 1 we display this ratio for different spectral indexes as a function of the height of peaks. It is easily oberserved that there is not any morphological preference. Moreover, when we only consider the highest peaks no strong trend is detected, even when they constitute a small sample.

The histogram of Figure 3 displays the distribution of oblate and prolate ellipsoids. This histogram and the ratios of Table 1, suggest that if an especific trend of shapes exists, in fact it behaves stochastically, and has no statistically relevance. Therefore, an intrinsic triaxial shape of density perturbations would be the most acceptable conclusion. An inspection of the distributions of Figure 3 also confirms that the results are not significantly dependent on the spectral index. The distribution of ellipticity and prolateness for all the models exhibit a slight tendency for oblate shapes. The mean values $<\epsilon>= 0.18 \pm 0.02$ and $<p>= (.13 \pm 0.003) \times 10^{-3}$ incorporate all the models. These values and the distribution of peaks in the $\epsilon - p$ plane show disagreement both with the theoretical predictions of BBKS and with the numerical ones of PH. In order to compare our results with those of these latter authors, it is necesary to assess the shapes by resorting to the Hessian matrix as indicated in section 3. Once the Hessian is diagonalized we take the ratio of axes defined by equation (10)

$$\left(\frac{\lambda_3}{\lambda_1}\right)^{1/2} = \frac{a_1}{a_3} = \frac{\mathrm{short-axes}}{\mathrm{long-axes}} = s, \tag{11}$$

$$\left(\frac{\lambda_3}{\lambda_2}\right)^{1/2} = \frac{a_2}{a_3} = \frac{\mathrm{middle-axes}}{\mathrm{long-axes}} = m. \tag{12}$$

Figure 4 is an example of the distribution of the ratio of axis in the $s - m$ plane. A visual judgement of such a distribution can easily lead us to wrongly conclude –as PH did– that a tendency of the peaks exists towards prolateness. In a $s - m$ diagram, a difference between a slight tendency to oblatness and a slight tendency to prolateness would be quite clear and not neglegible. The reason for this artificial discrepancy with our results is that PH did not consider the line which separates these two tendencies: $p = 0, \ \forall\ \epsilon$ in the interval $0 \leq \epsilon \leq 0.33$, i.e. all the eigenvalues which satisfy

$$\lambda_1 - 2\lambda_2 + \lambda_3 = 0, \tag{13}$$

or in terms of the ratio of axes

$$\frac{1}{s^2} - \frac{2}{m^2} + 1 = 0, \tag{14}$$

which gives the equation of the curve

$$s = \frac{m}{\sqrt{2 - m^2}}. \tag{15}$$

Shapes of Density peaks			
Spectral index n	1^{st}Realization (Inertia Tensor) $\Gamma = N_o/N_p$	2^{nd}Realization (Inertia Tensor) $\Gamma = N_o/N_p$	3^{th}Realization (Hessian) $\Gamma = N_o/N_p$
$n = 1$			
$\nu > 3.5$	110/105	112/111	104/101
$\nu > 2.5$	499/495	502/506	503/498
$\nu > 1$	3140/3144	3093/3092	3098/3101
$\nu > 0$	3283/3276	3337/3341	3305/3306
$n = 0$			
$\nu > 3.5$	93/95	113/109	107/102
$\nu > 2.5$	413/409	402/410	423/417
$\nu > 1$	2527/2512	2314/2321	2323/2321
$\nu > 0$	2687/2693	2498/2475	2548/2539
$n = -1$			
$\nu > 3.5$	70/68	73/76	69/72
$\nu > 2.5$	297/305	305/308	313/309
$\nu > 1$	1789/1797	1810/1797	1805/1812
$\nu > 0$	2057/2048	2023/2026	2063/257
$n = -2$			
$\nu > 3.5$	37/32	42/41	38/41
$\nu > 2.5$	153/158	165/163	157/155
$\nu > 1$	1071/1083	1112/1096	1086/1083
$\nu > 0$	1386/1394	1367/1356	1352/1343

Table 1. Ratio of the number of oblate objects to the number of prolate one, as a function of the spectral index n and the high of peaks. The used method is indicated.

When this function is included in the $s - m$ diagram of Figure (4), it is observed that the results of PH are also consistent with no tendency, perturbations are intrinsically triaxial as is further shown both by the distribution of the number of oblate and prolate shapes of Figure 5 and by the fourth column of Table 1. Going back to the results of BBKS one would wonder why there should be a *natural bias* for oblate shapes ?

4.2. Random generation of λ_i

The primordial density field is a Gaussian random field, i.e. it is generated by superposing a large number of Fourier modes with phase angles drawn at random from a uniform distribution in the interval $0-2\pi$. The shapes of density peaks, regarding them since the Hessian point of view, are the result of the curvatures of the density field on the three main axes. The eigenvalues of the Hessian matrix are therefore randomly generated numbers as well. The value of one does not affect the value of the other two. Under this assumption, it is no clear how a tendency towards oblateness can arise. It is also under this assumption that we have generated triplets of random numbers, without paying attention for them to fulfill the basic requirements of a cosmological density field; power law spectra, spectral index, etc. This has the goal of trying to reproduce our triaxiality distribution as a random process. This would indicate that the tendency infered from the BBKS results is a consequence of several analytical simplifications which hide the real triaxial shape of peaks.

We will call the triplets, eigenvalues,such that $0 < \lambda_i \leq 1$. Once they are put in an increasing order, we determine the triaxiality parameters through equation (5). The results of this are also displayed in the distribution of Figure 3 (in dashed line), which present no substantial differences with the distribution of shapes for a Gaussian random field obtained by using the inertia tensor and Hessian methods. Moreover, we can provide a meassure of how equal these two distributions are, in terms of their moments; mean $< p >$, standard deviation σ,

Fig. 3. Distribution of oblate and prolate shapes of peaks as a function of the spectral indexes, $n = 1, -2$, and hight $\nu > 0$. These distributions correspond with the cases of Table 1. The distribution in dashed line is a theoretical distribution of triaxiality, constructed by generation of random triplets λ_i (see Sec. 4).

Fig. 5. a) The same as Figure 4 for $n = -1$, but here the boundary curve (eq. 15) between the oblate and the prolate shapes is included. b) This distribution of oblate and prolate shapes, and fourth column of Table 1, show that the intrinsic shape of peaks is triaxial

Fig. 4. Distribution of peaks according with the ratio of axis, s and m, as presented by Peacock and Heavens (1985), where an artificial tendency towards prolate shapes is observed.

skewness s and kurtosis k. These moments characterize the observed asymmetry of the triaxiality distribution. For the Gaussian density field we obtain: $\bar{p} = 1.324 \times 10^{-2}, \sigma = 0.1189, s = 0.9091$ and $k = 0.9364$. Whereas for the distribution determined by the random triplets we get: $< p >= 8.53 \times 10^{-3}, \sigma = 0.1194, s = 0.9251$ and $k = 0.9635$. Therefore, it is quite likely that this two distributions are generated by the same stochastic process.

5. Conclusions

The present investigation underlines the nature of primordial density peaks and the dependence of the triaxiality distribution on the spectral index, for power law spectra. This numerical analisys explored the veracity of the PH and BBKS conclusions on these points.

For all the spectral indexes, we confirmed the BBKS result that high peaks tend to be more spherical than the lower ones. When the whole sample of peaks is considered, an intrinsic triaxial morphology is observed. This result solves a controversy concerning a natural initial bias towards prolateness shapes claimed by Heavens and Peacock (1985) and Dubinski (1992), or towards oblateness infered from the Bardeen *et al.* (1986) results. Moreover, the recent determinations on the intrinsic shape of elliptical galaxies by Fasano & Vio (1991) and Lambas *et al.* (1992), which resulted consistent with triaxial objects, suggest that the distribution of shapes survives the non-linear evolution.

We shall study the aspherical collapse of perturbations in the non-linear density field, investigating the relationship between the initial asphericity and the stability of properties of the distribution of collapse structures in the presence of a tidal field and in isolation

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